School Year 2003

Graduate School of Information Sciences
Tohoku University
Master's Course (First Two Year)
Entrance Examination Problems (March, 4)

The 9th Group of Subjects:

Economics

Note

Choose four Problems among the following eight Problems, and answer on the "Answer Sheet" (答案用紙).
Use only one Answer Sheet for each Problem. Clearly write the Problem number that you choose to answer in the box

問題番号	

which is given on the upper-left of each Answer Sheet.

東北大学大学院情報科学研究科平成15年度博士課程前期・入学試験問題（3月4日）
専門科目群第9　経済学群

Master's Course (First Two Year) Entrance Examination Problems
The 9th Group of Subjects: Economics

Problem1 Suppose a household has a utility function of the following form with respect to the consumption of two goods, x_1 and x_2.

$$u = 0.4\log x_1 + 0.6\log x_2$$

Letting y, p_1, and p_2 denote the income of a household and prices of two goods, respectively, answer to the following questions.

1 − 1　Derive the optimal consumption of two goods when $p_1 = 1$, $p_1 = 2$, and $y = 200$ (show the derivation process as well).

1 − 2　Derive the demand function of each good.

1 − 3　Taking advantage the results of 1 − 2, calculate the elasticities of consumption of each good with respect to the own price, another good's price, and income when $p_1 = 1$, $p_2 = 2$, and $y = 200$.

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Problem 2 Production function of a firm using two inputs, capital K and labor L, is represented as follows:

$$Y = K^{\alpha} + L^{\beta}$$

where Y is output, and $0 < \alpha < 1$, $0 < \beta < 1$. The prices of capital and labor are, respectively, denoted by r and w. Answer the following questions.

2 − 1　Derive the short-run cost function where the amount of capital is fixed.

2 − 2　Derive the log-run cost function where both inputs are variable.

2 − 3　Derive the supply function of a firm in the short-run perfectly-competitive market. (Let the market price be denoted by p).

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Problem 3 Suppose a household has the following quasi-linear utility function with respect to two goods, z_1 and z_2.

$$u = z_1 + 3\log z_2$$

Explain the properties of this utility function and draw the indifference curves so that they properly represent the properties.

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Problem 4

4–1 Explain the "quantity equation of money". If you use symbols, their definitions must be clarified.

4–2 Explain the "classical dichotomy", and discuss its relationship with the "quantity equation".

4–3 Explain the "LM curve", and discuss its relationship with the "quantity equation".

4–3 Explain the "liquidity trap", and discuss the effectiveness of fiscal and monetary policies under such circumstances.

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Problem 5 Answer the following questions about Nash equilibrium that is one of the most important equilibrium concepts in modern Economic Theory.

5 − 1　Consider an economic situation where n agents are interacting and affecting each other. The utility (or payoff) of each agent is represented by $u_i(x_1, x_2, \ldots, x_{i-1}, x_i, x_{i+1}, \ldots, x_n)$ where x_i is the sole decision variable of agent i. Give the definition of Nash equilibrium. You can assume that decisions are simultaneous and information is perfect and complete.

5 − 2　Explain what is Bayesian Nash equilibrium.

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Problem 6

An optimal consumption problem is given below. Answer the following questions.

$$\max_x u(x)$$
$$s.t.$$
$$p \cdot x = I$$

where $u(x)$ is the utility function, $x = (x_1, \ldots, x_n)$ is the consumption vector, $p = (p_1, \ldots, p_n)$ is the price vector, $p \cdot x$ is the inner product of p and x, and I is the income.

6 − 1　Write the Kuhn-Tucker conditions (including 'complementary slackness' condition) of the optimization problem. Use the variable λ for the Lagrange multiplier.

6 − 2　Assume $\lambda \neq 0$ at the solution. Show that the Lagrange multiplier λ represents the marginal utility of income.

6 − 3　The optimal solution , $x_j(p,I)$, is called Marshallian demand function, and the utility at the optimum, $v(p,I) = u(x(p,I))$, is called indirect utility function. Show that the following Roy's identity holds.

$$x_j(p,I) = -\frac{\partial v(p,I)/\partial p_j}{\partial v(p,I)/\partial I}, \quad j = 1, \ldots, n.$$

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Problem 7 Suppose that the density function of two variables, x and y, is given as follows.

$$\begin{aligned} f(x,y) &= k(x+y) \quad \text{for} \quad 0 \le x \le 1, 0 \le y \le 2 \\ &= 0 \quad \text{otherwise.} \end{aligned}$$

7–1 Determine the parameter k.

7–2 Calculate the expectation values, $E(x)$, $E(y)$, and $E(xy)$.

7–3 Show that the covariance of x, y is given by the expression, $\sigma_{xy} = E(xy) - E(x)E(y)$. Then determine its value.

7–4 When the variances of x and y are given by $\sigma_x^2 = 13/162$ and $\sigma_y^2 = 23/81$, respectively, calculate the variance $V(x+y)$ of $x+y$.

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Problem 8 The table below is the partial listing of populations of the top fifty cities of Japan based on the census for the year 2000.

1	Tokyo (Wards)	8134688	11	Saitama	1024053
2	Yokohama	3426651	12	Kitakyushu	1011471
3	Osaka	2598774	15	Sakai	792018
4	Nagoya	2171557	20	Kagoshima	552098
5	Sapporo	1822368	25	Himeji	478309
6	Kobe	1493398	30	Kawaguchi	460027
7	Kyoto	1467785	35	Oita	436470
8	Fukuoka	1341470	40	Hirakata	402563
9	Kawasaki	1249905	45	Machida	377494
10	Hiroshima	1126239	50	Asahikawa	359536

Let x and $G(x)$ denote the population of a city and its rank, respectively. Then the product is known to approximately satisfy $xG(x) = A$ (constant). This relation is called *Zipf's law* (or rank-size rule).

8–1 When we regard the population of *Tokyo Wards Area* as the constant A, calculate the population of the fiftieth largest city predicted by *Zipf's law*. Alternatively, if $x^\alpha G(x) = A$ holds instead of $xG(x) = A$, how large is the parameter α calculated from the population of Asahikawa?

8–2 Spell down a regression model that can be used to estimate, by an OLS method, $\alpha > 0$ and A, which most fit the fifty cities listed in the table.

8–3 When the sums of squares regarding the fifty cities are given as below, determine the parameters for the model stated above. In addition, calculate the unbiased variance of errors associated with the model.

$\sum_i G(x_i) = 1275.0$ $\quad \sum_i G(x_i)^2 = 42925.$ $\quad \sum_i x_i = .45561\times10^8$ $\quad \sum_i x_i^2 = .11377\times10^{15}$

$\sum_i \ln G(x_i) = 148.48$ $\quad \sum_i (\ln G(x_i))^2 = 479.65$ $\quad \sum_i \ln x_i = 669.65$ $\quad \sum_i (\ln x_i)^2 = 8991.3$

$\sum_i x_i G(x_i) = .65495\times10^9$ $\quad \sum_i \ln x_i \ln G(x_i) = 1959.2$ $\quad \sum_i G(x_i) \ln x_i = 16656.$ $\quad \sum_i x_i \ln G(x_i) = .91105\times10^8$

8–4 *Zipf's law* insists on $\alpha = 1$. Test $H_0 : \alpha = 1$ using the result obtained above. The single-sided rejection limits of t-distribution are: 2.58 at 0.5%, 2.33 at 1%, 1.96 at 2.5%, and 1.65 at 5%.

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Foreign Language（外国語）

Note

The two-page Problem Sheet, the one-page Answer Sheet, and the one-page empty sheet (for your free/convenient use) are included.
When the examination time started, you can separate these sheets if you like.

東北大学大学院情報科学研究科平成 15 年度博士課程前期・入学試験問題（3 月 4 日）

専門科目群第 9　経済学群

Master's Course (First Two Year) Entrance Examination Problems
The 9th Group of Subjects: English

1. Read the following statements about Cooperation in Economics, and summerize them about 200 words.

Cooperation in the economic tradition is mutual assistance between egoists. The archetypical example is Rousseau's stag-hunt: by coordinating their efforts, the two hunters more than double the returns of one isolated hunter; therefore each hunter can receive a positive profit from cooperation. Thus is cooperation justified (from the normative angle of rational pursuit of one's interest) and predicted (a positive statement about the behavior of selfish hunters).

That cooperation can be explained by the rational choice of self-interested parties, rather than by altruism, was most clearly stated in the celebrated passage from The Wealth of Nations: "In almost every other race of animals each individual, when it is grown up to maturity, is entirely independent, and in its natural state has occasion for the assistance of no other living creature. But man has almost constant occasion for the help of his brethren, and it is in vain for him to expect it from their benevolence only. He will be more likely to prevail if he can interest their self-love in his favour, and show them that it is for their own advantage to do for him what he requires of them. Whoever offers to another a bargain of any kind, proposes to do this. Give me that which I want, and you shall have this which you want, is the meaning of every such offer; and it is in this manner that we obtain from one another the far greater part of those good offices which we stand in need of. It is not from the benevolence of the butcher, the brewer, or the baker, that we expect our dinner, but from their regard to their own interest." (Smith [1784]).

The concept of Pareto optimality (also called "economic efficiency," or simply "efficiency") follows logically from this view of cooperation. An outcome x is Pareto optimal if the concerned agents cannot find another outcome y (one that they could achieve by mutual consent) such that every agent is better off in y than in x, or at least, in y no one is worse off and someone is better off (we say that outcome y is Pareto superior to outcome x). Thus if an outcome x is not Pareto optimal, it should not be the eventual outcome inasmuch as our agents can grab a cooperative opportunity by enforcing a Pareto superior outcome. Conversely, if outcome x is Pareto optimal, it cannot be dismissed by unanimous consent: x can only be rejected because it violates certain individual rights or certain principles of justice.

Pareto optimality is the one and only uncontroversial normative argument in economic theory (the New Welfare Economics, shortly after World War II, made this claim very forcefully; see Samuelson [1947]). One of the main tasks of economic analysis (some would say its only legitimate task) is to look for ways and means to promote a Pareto optimal outcome of the economy.

In some simple cases (involving a few agents who have complete knowledge of the cooperative opportunities, as in the stag-hunt story, or when I agree with my neighbor that we will keep an eye on each other's homes), it is realistic to postulate that efficiency must prevail among rational agents: they will cooperate if by doing so they both benefit. Coase [1960] provides the classical statement of this efficiency postulate. Note that the postulate, if it considerably simplifies the discussion of microeconomic behavior, does not leave the economist unemployed. For her skill is needed to work out the analytical consequences of Pareto optimality (including such facts as the equality of marginal rates of substitution and

other efficiency tests). Moreover, under the efficiency postulate the formation of coalitions and alliances cannot be dismissed: if the grand coalition captures all the cooperative opportunities, surely a smaller group of agents has no difficulty doing the same. This, in turn, raises a deep question: which efficient outcomes are stable when each and every coalition can cooperate as well?

Most actual markets for the allocation of goods and services involve a large number of agents who cannot directly communicate; the efficiency postulate, then, is not a realistic assumption, because it rests on direct agreement of all the participants in the market. Economists have argued since Adam Smith that a price signal is an efficient decentralization device for the cooperative organization of the market. The advantage of the price system over face-to-face bargaining is that after learning the price, an agent need not further communicate with other participants in the market. Taking the price as given, he acts selfishly (buying or selling as much as pleases him); the result of these uncoordinated selfish decisions is, miraculously enough, a Pareto optimal outcome. Thus everyone in the market is "led by an invisible hand to promote an end (efficiency) which was no part of his intention" (Smith [1784]). The invisible-hand metaphor captures the gist of the competitive price mechanism. The model of competitive exchange, still the unchallenged masterpiece of economic theory, is the prototype of a cooperative institution which satisfies the economists (and many others).

Source : Moulin, H., Cooperative Microeconomics, Princeton University Press, 1995.