Administered in Fiscal 2003

Graduate School of Information Sciences
Tohoku University
Master's Course (First Two Years)
Entrance Examination Problems (March, 2)

The 7th Group of Subjects:

Social Sciences

Note

Choose three Problems among the following eighteen Problems, and answer on the "Answer Sheet" (答案用紙).
Use only one Answer Sheet for each Problem. Clearly write the Problem number that you choose to answer in the box

問題番号	

which is given on the upper-left of each Answer Sheet.

平成15年度実施

東北大学大学院情報科学研究科博士課程前期・入学試験問題(3月2日)

専門試験科目群第7・社会科学群

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Master's Course (First Two Year) Entrance Examination Problems (March, 2)

The 7th Group of Subjects: Social Sciences

Problem E–1 *Suppose a consumer has a utility function of the following form with respect to two goods,* x_1 and x_2;

$$u = x_1 + 2\log x_2$$

Letting p_1 and p_2 be prices of the two goods, respectively, and I the income of a consumer, answer to the following questions.

1–1 Seek for the optimal consumption bundle when $p_1 = 10$, $p_2 = 3$, and $I = 300$.

1–2 Derive the demand function of each good.

1–3 Explain the possible properties of good 2. Then draw the indifference curves so that the properties of good 2 are represented.

平成15年度実施

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Problem E–2 *Suppose the market demand function is represented as* $p = 100 - y$, *and* the market supply function as $p = 10 + 2y$ where p is the market price and y the quantity. Answer the following questions.

2–1 Seek for the equilibrium price and quantity when the market is perfectly competitive.

2–2 Seek for the equilibrium price and quantity when a supplier is a monopolist.

2–3 Compare the two equilibria from the viewpoints of 'efficiency' of resource allocation and 'equality' of income distribution.

平成15年度実施

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Problem E–3 'Agglomeration economies' is considered as an important factor for city formation and growth of cities. Explain why.

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Problem E–4

4–1 Suppose the capital investment during the year t and annual capital depreciation rate are given by I_t and ρ, respectively. When we denote the capital asset at the end of year t by K_t, show the *difference equation* regarding K_t. How the equation must be altered when K_t gives the value at the beginning of year t, rather than the end?

4–2 In the following, we choose K_t to denote the year-end value. Provide the expression of K_t based on the initial asset K_0 at the end of year 0.

4–3 When the annual interest rate is fixed at r, calculate the present value $PV(K_t)$ of K_t evaluated at the initial year ($s = 0$). Also calculate the present value $PV(I_t)$ of the total investments made during years $s = 1 \ldots t$.

4–4 In the above, the annual investments I_t are regarded as real. In reality, the capital commodity price fluctuates. When we denote the price in the year t by P_t, provide the expression for the *real present value* of total investments $RPV(I_t)$ based on the price level in year 0. Give the proper name for the index P_t/P_0.

4–5 The Paasche and Laspeyres indices provide the typical expressions for the indices like P_t/P_0. Explain briefly the differences between those two types of indices. Why it is difficult to calculate such indices for a long term?

平成16年度

東北大学大学院情報科学研究科博士課程前期・入学試験問題(3月2日)

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Problem E–5 Consider the following two regressions.

$$y_i = a + bx_i + u_i, \tag{3}$$

$$y_i^0 = \beta x_i^0 + u_i, \tag{4}$$

where x_i^0 and y_i^0 are defined as $x_i^0 = x_i - \bar{x}$ and $y_i^0 = y_i - \bar{y}$, respectively, using the means $(\bar{x}, \bar{y})$ of (x_i, y_i). u_i represents the disturbance that follows the normal distribution, $N(0, \sigma^2)$.

5–1 Show the normal equations for (1), and calculate the estimators $\hat{a}$ and $\hat{b}$ for the parameters.

Note: You may choose either expressions based on the elements or the vectors. When you choose the latter form, you should define the vectors before using them.

5–2 Describe the estimators $(\hat{a}, \hat{b})$ in terms of the variances, $Var(x), Var(y)$, and the covariance, $Cov(x, y)$, of x_i and y_i.

5–3 Show the normal equation for (2), and confirm that the estimator $\hat{\beta}$ coincides with $\hat{b}$ obtained above.

5–4 Calculate the *expected value* and the *variance* of the estimator $\hat{\beta}$.

平成15年度実施

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問題E–6 Consider a firm producing one output by two factor inputs of capital and labor. The firm has the following production function.

$$y = KL$$

where y is the output level, and K and L are input levels of capital and labor, respectively. The factor prices of capital and labor are given by w_K and w_L, respectively. Answer the following questions.

6–1 Consider the 'short-run' production where the capital input level is fixed at K_0. Derive the short-run cost function of the firm, $SC(K_0, y)$.

6–2 Consider the 'long-run' production where the firm can determine all factor input levels (namely, the capital level K and the labor level L). Derive the long-run cost function, $LC(y)$.

6–3 Show that the following equation (which is called 'the envelope theorem') holds.

$$\frac{dLC(y)}{dy} = \frac{\partial SC}{\partial y}(K(y), y)$$

where $K(y)$ is the capital input level that minimizes the long-run cost (namely, $K(y)$ is the optimal capital level).