School Year 2003

Graduate School of Information Sciences
Tohoku University
Master's Course (First Two Year)
Entrance Examination Problems (September, 3)

The 9th Group of Subjects:

Economics

Note

Choose four Problems among the following eight Problems, and answer on the "Answer Sheet" (答案用紙).
Use only one Answer Sheet for each Problem. Clearly write the Problem number that you choose to answer in the box

問題番号	

which is given on the upper-left of each Answer Sheet.

東北大学大学院情報科学研究科平成 15 年度博士課程前期・入学試験問題（9 月 3 日）
専門科目群第 9　経済学群

Master's Course (First Two Year) Entrance Examination Problems
The 9th Group of Subjects: Economics

Problem1 Suppose a consumer has a utility function of the following form with respect to the consumption of the two goods, x_1 and x_2.

$$u = x_1^{\alpha} x_2^{\beta} \qquad (\alpha, \beta > 0) \tag{1}$$

Let the income of a consumer be $y = 120$.

1 − 1　Derive the optimal consumption of two goods when prices of two goods are $p_1 = 2$ and $p_2 = 5$, respectively.

1 − 2　How are the optimal consumptions changed when the income is increased to $y = 180$?

1 − 3　Draw the indifference curves for preference in (1) so that the feature of preference can be identified graphically.

1 − 4　Letting the two prices and income level be denoted by p_1, p_2, and y, respectively, derive the demand curve for good 1.

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Problem 2 Suppose a firm produces output y by using a variable input z_1 and a fixed input z_2, and its production function is represented as

$$y = z_1^{0.5} + z_2^{0.5}$$

Prices of z_1 and z_2 are $w_1 = 5$ and $w_2 = 10$, respectively, and q denotes the price of output.

2 − 1　Derive the long-run and short-run cost functions of this firm, respectively.

2 − 2　Derive the short-run average and marginal cost functions, respectively.

2 − 3　Derive the supply curve of this firm and draw it when it operates in the short-run competitive market.

Problem 3 Suppose the market demand curve of a commodity is represented as

$$D = 200 - 0.4p$$

where D is the amount of demand and p is the price paid by demanders. On the other hand, the supply curve of firms as a whole operating in the market is represented as

$$S = 20 + p$$

where S is the amount of supply.

3 − 1　Derive the equilibrium price and quantity traded when the market is perfectly competitive.

3 − 2　Derive the equilibrium price and quantity traded when the market is monopolistic.

3 − 3　Suppose the 5%-consumption tax (5% of sales) as indirect tax is introduced in the system. How is the supply curve represented under the 5%-consumption tax?

3 − 4　In what condition is this consumption tax shifted entirely to firms?

Problem 4

4–1 The existing industrial capital can be described in terms of a difference equation, $K = (1-\delta)K_{-1} + I$, where K, I, and δ are the capital stock, investment, and capital depreciation rate, respectively. The subscript -1 indicates the variable in the previous period.

Explain why "over-heat" in economy is not desirable by referring to the potential rate of economic growth. If necessary, you may cite a linearly homogenous production function.

4–2 Suppose the marginal propensity to consume c and the tax rate t are constant, and consider an IS-LM model,

$$Y = c(1-t)Y + (\bar{I} - bi) + G \quad \text{and} \quad M/P = kY - hi,$$

where $Y, i, \bar{I}, G$, and M/P are the income, interest rate, fixed part of investment, government expense, and real money supply, respectively. Other symbols not mentioned are positive parameters.

Explain why the tax works as a built-in stabilizer to the economy. You may refer to the effect of increase in $\bar{I}$ on income.

4–3 Briefly explain what is the stabilization policy. Discuss the roles and limitations of fiscal policies as a tool for economic stabilization.

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Problem 5 Consider a one-dimensional locational space [0,1]. Two independent shops, A and B, are about to determine their location on this space. They sell the same commodity without any production cost. The number of customers of the commodity is n and they are uniformly distributed in this one-dimensional space. Each customer purchases one commodity inelastically from a shop that is closest to his/her location. When purchasing the commodity from the shop which has x distance from his/her location, the customer has to spend the travel cost of cx^2. Answer the following questions.

5－1　The selling prices are now same and fixed for the both shop, p. Find the equilibrium location of the two shops.

5－2　Assume that the locations of Shop A and Shop B are fixed at the location 0 and location 1/2, respectively, and are about competing over selling prices. Find the equilibrium prices.

5－3　Assume that the two shops play a two stage game where they first determine the shop locations and then determine the prices. Find the equilibrium.

Problem 6

Answer the following questions about the matrix A given below.

$$A=\begin{bmatrix} 3 & 0 & 0 \\ -4 & 6 & 2 \\ 16 & -15 & -5 \end{bmatrix}$$

6 − 1　Check if the matrix A is 'regular'.

6 − 2　Find the eigenvalues of the matrix A.

6 − 3　Find the eigenvectors of the matrix A.

平成15年度

東北大学大学院情報科学研究科博士課程前期・入学試験問題(9月3日)

専門科目群第9・経済学群

Problem 7 The upper limit of the probability that the difference between random variable X and its mean μ exceeds a constant c is given by the Chebychev's inequality,

$$P(|X-\mu| \geq c) \leq \sigma^2/c^2.$$

7–1 Using the probability density $f(X)$ of X, describe the probability $P(|X-\mu| \geq c)$ and the variance σ^2, respectively, in terms of definite integrals.

7–2 Prove the Chebychev's inequality using the above integrals.

7–3 Suppose $\bar{X}$ denotes the sample mean calculated from n samples, $X_1, X_2, \ldots, X_n$, which are *mutually independent*. Show that its expectation and variance become $E(\bar{X}) = \mu$ and $V(\bar{X}) = \sigma^2/n$, respectively.

7–4 Briefly explain the *law of large numbers* using the Chebychev's inequality concerning $\bar{X}$.

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専門科目群第9・経済学群

Problem 8 When we regress the real investment I (\$ trillion) on the real GDP Y (\$ trillion) and interest rate i (%), based on the U.S. annual data between 1968 and 1982, we obtain the following result.

$$I_t = -72.64 + 0.2359Y_t - 3.708i_t \quad R^2 = 0.7775, \quad DW = 1.475,$$

where R^2 and DW are the R-squared and the Durbin-Watson statistic, respectively.

8–1 Calculate the adjusted R-squared (concerning the degree of freedom). Discuss why people attach importance to the adjusted coefficients when choosing a model. Also explain the inconsistency (if any) associated with the adjusted coefficients.

8–2 Calculate the residual sum of squares S_E and the standard error $\hat{\sigma}$ of the model when the total sum of squares is $S_T = 16293.18$.

8–3 The diagonal elements of the inverted cross-product matrix $(X'X)^{-1}$ are obtained as follows; $a_{YY} = 7.859E - 6$ for Y and $a_{ii} = 0.02554$ for i. Calculate the t-statistics for the parameters associated with these two variables, and test the null hypothesis $\mathrm{H_o}: \quad \beta_k = 0$.

The relevant critical limits (single-sided) are given by 2.179 at 2.5% and 3.055 at 0.5%.

8–4 Are you satisfied with the above model? Point out if you find some statistical problems, and discuss the possible improvements.

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Foreign Language (外国語)

Note

The two-page Problem Sheet, the one-page Answer Sheet, and the one-page empty sheet (for your free/convenient use) are included.
When the examination time started, you can separate these sheets if you like.

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The 9th Group of Subjects: English

1. Read the following and summarize them about 200 words.

If small events in history had been different, would the pattern of cities we have inherited be different in any significant way? Could different "chance events" in history have created a different formation of urban centers than the one that exists today?

To a great degree, cities form around and depend upon clusters of industry, so that without doing too much injustice to the question we can ask whether the patterns of location of industry follow paths that depend upon history. The German Industry Location School debated this question in the earlier part of this century, but it was never settled conclusively. Some theorists saw the spatial ordering of industry as preordained—by geographical endowments, shipment possibilities, firms' needs, and the spatial distribution of rents and prices that these induced. In their view, history did not matter: the observed spatial pattern of industry was a unique "solution" to a well-defined spatial economic problem. Therefore, early events in the configuration of an industry could not affect the result. Others saw industry location as path-dependent—as an organic process with new industry laid down upon and very much influenced by inherited, locational patterns already in place. Again geographical differences and transport possibilities were important, but here the main driving forces were agglomeration economies the benefits of being close to other firms or to concentrations of industry. In this view, early firms arriving by "historical accident" might put down in locations they were attracted to for geographical reasons. Later firms might be attracted to these same places by the presence of these early locators, rather than geography. Still later firms might be attracted in turn by their presence. The industry ends up clustered in the early-chosen places. But this spatial ordering is not unique: a different set of early choosers could have steered the locational pattern into quite a different outcome, so that settlement history would be crucial.

These two viewpoints—determinism versus history dependence, or "necessity" versus "chance"—are echoed in current discussions of how modern industrial clusters have come about. The determinism school, for example, would tend to see the electronics industry in the United States as spread over the country, with a substantial part of it in Santa Clara County in California (Silicon Valley) because that location is close to Pacific sources of supply and because it has better access there than elsewhere to airports, skilled labor, and advances in academic engineering research. Any "small events" that might affect location decisions are overridden by the "necessity" inherent in the equilibration of spatial economic forces; and Silicon Valley is part of an inevitable result. Historical dependence, on the other hand, would see Silicon Valley and similar concentrations as largely the outcome of "chance." Certain key persons—the Packards, the Varians, the Shockleys of the industry happened to set up near Stanford University in the 1940s and 1950s, and the local labor expertise and interfirm markets they helped to create in Santa Clara

County made subsequent location there extremely advantageous for the thousand or so firms that followed them. If these early entrepreneurs had had other predilections, Silicon Valley might well have been somewhere else. In this argument, "historical chance" is magnified and preserved in the locational structure that results.

Although the historical dependence-agglomeration argument is appealing, it has remained problematical. If history can indeed steer the spatial system down different paths, there are multiple "solutions" to the industry location problem. Which of these comes about is indeterminate. In the 1920s, analysts could not cope with this difficulty, and the historical chance argument did not gain enough rigor to become completely respectable.

Source: Arthur, W. B., Increasing Returns and Path Dependence in the Economy, The University of Michigan Press, 1994.